DAYSTAR UNIVERSITY

COURSE NAME:FUNDAMENTALS OF ACTUARIAL SCIENCE 2

COURSE CODE :ACM 204

INSTRUCTIONS: ATTEMPT QUESTION 1 (COMPULSORY) AND ANY OTHER TWO

QUESTION ONE [30 MARKS]

1a). At a certain company, the probability of each employee leaving during any given year is 5 %, independent of other employees. Those who remain with the company for 25 years are given kshs.10 million, What is the expected present value of this payment to a new starter assuming an interest rate of 7% and ignoring the possibility of death [3 marks]

b).Calculate the standard deviation of the present value [2 marks]

2.Calculate the expected present value and variance of the present value of a term assurance of 1 payable immediately on death for a life aged exact 40 if death occurs within 30 years.

Basis:4% per annum,A1967-10 ultimate mortality [5 marks]

3.Define :

i. surrender value [2 marks]
ii. paid up policy value [2 marks]
iii. Reserve [2 marks]

4.A life aged exactly 30 buys a 15 year endowment assurance policy with a sum assured of kshs.100,000 payable on maturity or at the end of year of earlier death. Level of premiums are payable monthly in advance .Calculating the monthly premium, assuming A1967-70 mortality at 4 % interest. Ignore the expenses. [6 marks]

5. Given that ${}_t p_{30}=\frac{7800-70t-t^2}{7800}$, $0 \le t \le 60$ [5marks]

Calculate the exact value of $q_{50} - u_{50}$

6. Calculate ${}_{10/}a_{30}$ and $\ddot{a}^{4}_{50}$, $a^{(2)}_{20:10}$based on A1967-70 ultimate mortality at 6% [3 marks]

QUESTION 2: [20 MARKS]

a).i)State the conditions under which the prospective and the retrospective reserves will be equal [2 marks]

ii. A whole life annuity is issued to a life aged x. The annuity is purchased by a single premium and the benefit of 1 is payable at the beginning of every year throughout life. Show that net prospective and retrospective reserves are equal [10 marks]

b)Ten years ago a life office issued a 20 year endowment assurance without profits to a person aged 35.The sum assured is $10,000 payable at the end of year of death or on survival for 20 years, and premiums are payable annually in advance. Using A1967-70 ultimate 6 % and ignoring expenses, Calculate

i. The annual premium [4 marks]
ii. The reserve ,assuming that premium now due has been paid [4 marks]

QUESTION 3 :[20 MARKS]

A)Find the single premium for a 10 year term assurance with the sum assured for $ 500 for a life aged 30 on the basis of the !967-70 select table at 4 %.The premium is paid to be returned with the sum insured in the event of a claim. Expenses are 3% of the sum assured with continuing administrative costs of $ 1 each year [5 marks]

b) Calculate on the basis of A1967-70 ultimate tables the annual premium for a 25 year endowment assurance on a life aged 40 allowing for the initial expenses of 4 % of the sum assured, renewal expenses of including renewal commission of 5% of each premium including 25% each year for $100 sum assured and the initial commission of 40% of the first year gross premium [6 marks]

c)i. Find the annual premium, halving after 15 years for a whole life assurance for a select life aged 35 on the basis of the A1967-70 table of mortality at 4 % [4 marks]

ii. To what value does the above premium reduce to after 15 years [1 mark]

d .Calculate the premium values using the commutation functions of A1967-70 table at 4 % interest

i. $P_{[30]:\overline{10}|}$ [2 marks]

ii. ${}_5P_{40}$ [2 marks]

QUESTION FOUR [20 MARKS]

a)Given that:

V_i -waiting time of the ith life in the able state

W_i - waiting time of the ith life in the ill state

S_i -number of transitions able to ill in the ith life

R_i - number of transitions ill to able in the ith life

D_i - number of transitions able to dead in the ith life

U_i - number of transitions ill to dead in the ith life

The likelihood function is :

$$L(u,\upsilon,\sigma,\rho) = e^{-[\mu+\sigma]v} e^{-[\upsilon+\rho]w} \mu^d \upsilon^u \sigma^s \rho^r$$

i.Find the maximum likelihood estimator for the parameters μ,υ,σ,ρ [10 marks]

ii. During a study into rates of sickness :

2500 healthy lives fell ill and 2490 ill lives recovered.80 healthy lives died and 170 sick lives died. For the whole group the period of health and illness totaled 35,600 and 5,400 years respectively. Estimate the annual forces of transition between the states able, ill and dead [5 marks]

b)State the assumptions underlying the single decrement model and also the probability of survival in terms of transition intensity [5 marks]

QUESTION FIVE [20 MARKS]

a. Show that $q_x = \dfrac{m_x}{1+\dfrac{1}{2}m_x}$ [5 marks]

b. For many years a company has recruited ,uniformly over each year,200 employees on their 20th birthdays and a fixed number of additional employees on their 25th birthdays. Mortality follows English Life table no.-12 males. Employees may retire on their 60th or 65th birthdays, and one third of employees reaching their 60th birthdays retire on that date. Employees leave the company only through death or retirement and the total number of employees is 10,000.Find the total number of new recruits each year [5 marks]

c.For a certain population $l_x = 10{,}000(121 - x)^{\frac{1}{2}}$ [6 marks]

Find u_x, q_x and the probability that a life aged 0 will die between ages 21 and 40

d.Using English life table no.12 –male calculate

i. $\overset{0}{e}_{2:10}$ [2 marks]

ii. T_{15} [2 marks]