DAYSTAR UNIVERSITY

**SCHOOL OF SCIENCE, ENGINEERING AND HEALTH**
**DEPARTMENT OF SCIENCE AND ENGINEERING**

**MATH 121A: DIFFERENTIAL CALCULUS**

**FINAL EXAMINATION**
**JANUARY SEMESTER, 2012**

**INSTRUCTIONS TO CANDIDATES**

I. ***Answer question ONE and any other TWO questions***
II. ***Points will be awarded for clear and concise working, and slovenly done work will be penalized***
III. ***Symbols have their usual meaning.***
IV. ***Other than question 1, all other questions carry equal points***
V. ***Mobile phones are not allowed in the examination hall, and cannot therefore be used as calculators.***
VI. ***The maximum possible points that can be earned in this paper is 60.***

**QUESTION ONE (COMPULSORY – 24 MARKS)**

I. State the domain of the following function

$$f(x) = \frac{x^3 + 2x^2 - 1}{3 - 5x}$$ **1mk**

II. Where is the following function discontinuous?

a. $f\left(x\right) = \begin{cases} \dfrac{1}{x^2} & if \quad x \neq 0 \\ 1 \ if \ x = 0 \end{cases}$ **1mk**

III. Use the laws of limits to find:

a. $\lim_{t \to 0} \dfrac{\sqrt{t^2 + 9} - 3}{t^2}$ **3mks**

b. $\lim_{x \to \infty} \dfrac{\sqrt{2x^2 + 1}}{3x - 5}$ **3mks**

IV. Use the first principle formula to evaluate $f'(x)$ given:

$$f(x)=\frac{1}{2}x^2-15$$ **3mks**

V. Use logarithmic differentiation to find $y'$ given :

$$y=x^2\sin x$$ **3mks**

VI. Use implicit differentiation to find $y'$ given:

$$x^2=\frac{x-y}{x+y}$$ **3mks**

VII. The area A of a circle is related to its diameter by the equation:

$$A=\frac{\pi}{4}D^2$$

Determine the rate of change of area with respect to the diameter when the diameter is 10m. **3mks**

VIII. Find the equation of the tangent to the curve $x^2-xy+y^2=7$ at $(-1,2)$ **4mks**

## QUESTION TWO (18 MARKS)

I. Differentiate:

$$r=\frac{(\theta-1)(\theta^2+\theta+1)}{\theta^3}$$ **3mks**

II. Use the laws of limits to find:

a. $\lim_{x\to 1}\frac{x^3-1}{x^2-1}$ **3mks**

b. $\lim_{h\to 0}\frac{(4+h)^2-16}{h}$ **3mks**

III. Find $\frac{\partial z}{\partial y}$ and $\frac{\partial z}{\partial x}$ given that $z=\frac{2x-y}{x+y}$ **6mks**

IV. One of the quantities of interest in thermodynamics is compressibility which is given by $\beta = -\frac{1}{v}\frac{dV}{dP}$ where V is volume and P is pressure. The volume V(in cubic metres) of a sample of air at $25^\circ C$ was found to be related to the pressure p (in kilopascals) by the equation $V = \frac{5.3}{P}$. Find compressibility when $P = 50\,kPa.$ **3mks**

## QUESTION THREE (18 MARKS)

I. Use implicit differentiation to find $\frac{dy}{dx}$ given:

a. $x^2y^2 + x\sin y = 4$ **3mks**

b. $x^3 + x^2y + 4y^2 = 6$ **3mks**

II. Use logarithmic differentiation to find $y'$ given:

$$y = \frac{1}{xe^x \sin x}$$ **4mks**

III. The position of a particle is given by the equation:

$$s = f(t) = 4t^3 - 9t^2 + 6t + 2$$

where t measured in seconds and s in meters.

Find:

i. The velocity and the acceleration after t seconds? **5mks**

ii. The velocity and the acceleration after 2seconds? **2mks**

iii. When is the particle at rest? **1mk**

## QUESTION FOUR (18 MARKS)

I. Use chain rule to differentiate the following function:

$$y = \left(\frac{\sqrt{x}}{1+x}\right)^2$$ **3mks**

II. The displacement of two particles on a co-ordinate line is given by

$s_1 = 3t^3 - 12t^2 + 18t + 5$ and $s_2 = -t^3 + 9t^2 - 12t$. At what time do the two particles have the same velocity? **3mks**

III. Use the first principle formula to evaluate $f'(x)$ given:

$$f(x) = \cos x$$ **3mks**

IV. The parametric equations of a curve are $x = \cos 2\theta$ and $y = 1 - \sin\theta$. Find $\frac{dy}{dx}$. **3mks**

V. If $x^2 + y^2 - z^2 = 2x(y+z)$ is an implicit function, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. **6mks**

## QUESTION FIVE (18 MARKS)

I. Use quotient rule and any other relevant rule to find $y'$ given:

a. $y = \frac{(x+2)(x^3-6)}{x^3-5}$ at $x = -2$. **4mks**

b. $y = \frac{x \sin x}{1 + \cos x}$ at $x = \frac{\pi}{4}$ **5mks**

II. Consider the curve:

$$y = \frac{x^3}{3} - \frac{x^2}{2} - 2x + 5$$

a. Find the stationary points. **3mks**
b. Test their nature. **3mks**
c. Sketch the curve. **3mks**