



SCHOOL OF SCIENCE, ENGINEERING AND HEALTH
DEPARTMENT OF SCIENCE AND ENGINEERING

MATH 221A: INTEGRAL CALCULUS

FINAL EXAMINATION
JANUARY SEMESTER, 2012

INSTRUCTIONS TO CANDIDATES

- I. Answer question ONE and any other TWO questions*
- II. Points will be awarded for clear and concise working, and slovenly done work will be penalized*
- III. Symbols have their usual meaning.*
- IV. Other than question 1, all other questions carry equal points*
- V. Mobile phones are not allowed in the examination hall, and cannot therefore be used as calculators.*
- VI. The maximum possible points that can be earned in this paper is 60.*

QUESTION ONE (COMPULSORY – 24 MARKS)

- I. Use substitution method to integrate:
 - a. $\int x^3 \cos(x^4 + 2) dx$ **3mks**
 - b. $\int_1^e \frac{\ln x}{x} dx$ **4mks**
- II. Use integration by parts to evaluate:
 - a. $\int \ln x dx$ **3mks**
 - b. $\int x^2 e^x dx$ **5mks**
- III. Integrate the following trigonometric integrals:
 - a. $\int \sin^5 x \cos^2 x dx$ **5mks**
- IV. By substituting $u = \sqrt{x} - 4$ evaluate the definite integral
$$\int_3^5 \frac{\sqrt{x} - 4}{\sqrt{x}} dx$$
 4mks

QUESTION TWO (18 MARKS)

- I. Evaluate: $\int e^x \sin x dx$ **6mks**
- II. Use the partial fractions or otherwise to prove
- $$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$$
- 4mks**
- And use the formula to evaluate:
- $$\int \frac{1}{x^2 - 64} dx$$
- 2mks**
- III. Find the length of the arc of the semi cubical parabola $y^2 = x^3$ between the points (1,1) and (4,8). **6mks**

QUESTION THREE (18 MARKS)

- I. Sketch the region and find the area bounded by the parabola $y = 2 - x^2$ and the line $y = x$. **8mks**
- II. Use Simpson's Rule with $n = 5$ to approximate the integral $\int_0^1 e^{x^2} dx$. **5mks**
- III. Use the trapezoidal rule with $n = 4$ to estimate

$$\int_1^2 x^2 dx$$

Compare the estimate with the exact value of integral. **5mks**

QUESTION FOUR (18 MARKS)

- I. Use integration of rational functions by partial fractions to evaluate:

$$\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} dx$$

9mks

- II. Find the centre of mass of the region bounded by the line $y = x$ and $y = x^2$. **9mks**

QUESTION FIVE (18 MARKS)

I. Evaluate: $\int \frac{x+3}{(x^2+6x)^2} dx$ **3mks**

II. Use integration of rational functions by partial fractions to evaluate:

$$\int \frac{2x+3}{(x+1)^2} dx$$
 5mks

III. Find the length of the curve:

$$y = \ln(\cos x), 0 \leq x \leq \frac{\pi}{4}$$
 10mks