DAYSTAR UNIVERSITY

SCHOOL OF SCIENCE, ENGINEERING AND HEALTH
DEPARTMENT OF SCIENCE AND ENGINEERING

MAT 223: DISCRETE MATHEMATICS

FINAL EXAMINATION
JANUARY 2012 SEMESTER

INSTRUCTIONS:

Answer question one and any other two questions
Symbols have their usual meaning, unless stated otherwise.

QUESTION ONE: COMPULSORY - 24 MARKS

a. Determine $f(2), f(3), f(4)$ and $f(5)$ if f is defined recursively by $f(0) = f(1) = 2$ and $f(n+1) = f(n) + f(n-1)$. **4 marks**

b. Using a direct proof, show that the sum of two odd integers is even. **3 marks**

c. Let *p, q,* and *r* be the following propositions:-

p: Hyenas have been seen in the area.
q: Hiking is safe on the trail.
r: Berries are ripe along the trail.

Write the following propositions using *p, q,* and *r* and logical connectives:-

i. If berries are ripe along the trail, hiking is safe if and only if Hyenas have not been seen in the area. **2 marks**

ii. For hiking on the trial to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for Hyenas not to have been seen in the area. **2 marks**

d. Evaluate each of the following expressions:-

i. $(11011 \vee 01010) \wedge (10001 \vee 11011)$ **2 marks**

ii. $(01010 \oplus 11011) \vee 01000$ **2 marks**

e. Let $p(m,n)$ be the statement *"m* divides *n"*, where the domain for both variables consists of all positive integers. Determine the truth values of the following statements.

i.	$\forall m \forall n\, p(m,n)$	**1 mark**
ii.	$\exists n \forall m\, p(m,n)$	**1 mark**
iii.	$\forall n\, p(1,n)$	**1 mark**

f. Suppose that $A = \{a,b,c\}$, $B = \{x,y\}$ and $C = \{0,1\}$. Evaluate each of the following:-

i.	$A \times B \times C$	**2 marks**
ii.	$B \times B \times B$	**2 marks**

g. Given that $A = \{1,2,3,4,5\}$ and $B = \{0,3,6\}$, find:-

i.	$A \cup B$	**1 mark**
ii.	$B - A$	**1 mark**

QUESTION TWO – 18 MARKS

a. The following is a "proof" that $2 = 1$:-

	Step	Reason
1.	$a = b$	Given
2.	$a^2 = ab$	Multiply both sides of 1 by a
3.	$a^2 - b^2 = ab - b^2$	subtract b^2 from both sides of 2
4.	$(a-b)(a+b) = b(a-b)$	Factor both sides of 3
5.	$a + b = b$	Divide both sides of 4 by $(a-b)$
6.	$2b = b$	Replace *a* by *b* in 5 because $a = b$ and simplify
7.	$2 = 1$	Divide both sides of 6 by *b*.

Identify the error in the "proof", if any. **2 marks**

b. Prove by contradiction that $\sqrt{7}$ is irrational. **5 marks**

c. Show that if n is a positive integer, then n is even if and only if $7n + 4$ is even. **4 marks**

d. Let $P(n)$ be the statement that $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$ for the positive integer n. What is the statement $P(1)$? **2 marks**

e. Use mathematical induction to show that $\sum_{j=0}^{n} ar^j = \frac{ar^{n+1} - a}{r-1}$. **5 marks**

QUESTION THREE – 18 MARKS

a. Suppose that the universal set is $U = \{0,1,2,3,4,...,10\}$,

   i. Express each of the following sets with bit strings

      1. $\{1,3,8,10\}$ **1 mark**
      2. $\{3,4,5\}$ **1 mark**

   ii. Find the set specified by the following bit strings:-

      1. 1111001111 **1 mark**
      2. 1000010001 **1 mark**

b. Given that *A*, *B*, *C*, and *D* are sets, use a ***membership table*** to determine if $(A \oplus B) \oplus (C \oplus D) = (A \oplus D) \oplus (B \oplus C)$ **8 marks**

c. Let $A_i = \{1,2,3,....,i\}$ for $i = 1,2,3,...$. Find:-

   i. $\bigcup_{i=1}^{n} A_i$ **3 marks**

   ii. $\bigcap_{i=1}^{n} A_i$ **3 marks**

QUESTION FOUR – 18 MARKS

a. In Asynchronous Transfer Mode (ATM), data are organized into cells of 53 bytes. How many ATM cells can be transmitted in 1 minute over a connection that transmits data at the rate of 500 kilobits per second? **3 marks**

b. Given an example of a function from $\mathbb{N}$ to $\mathbb{N}$ that is

   i. One-to-one but not onto **1 mark**
   ii. Onto but not one-to-one **1 mark**
   iii. Both onto and one-to-one **1 mark**
   iv. Neither one-to-one nor onto. **1 mark**

c. Find the value of:-

a. $\left\lceil \frac{3}{4} \right\rceil$ **1 mark**

b. $\left\lfloor \frac{1}{2} + \left\lceil \frac{3}{2} \right\rceil \right\rfloor$ **1 mark**

c. $\left\lfloor \frac{1}{2} \times \left\lfloor \frac{5}{2} \right\rfloor \right\rfloor$ **1 mark**

d. Draw the graph of $f(x) = \left\lfloor 2\left\lceil \frac{x}{2} \right\rceil + \frac{1}{2} \right\rfloor$ **4 marks**

e. Evaluate the following:

i. $\sum_{i=0}^{2}\sum_{j=0}^{3} i^2 j^2$ **2 marks**

ii. $\sum_{i=1}^{3}\sum_{j=0}^{2} (i - j)$ **2 marks**

QUESTION FIVE – 18 MARKS

a. Find the product of 1011_2 and 110_2 **2 marks**

b. Convert each of these integers from binary notation to hexadecimal notation:-

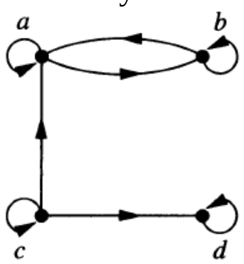
i. 11110111 **2 mark**

ii. 101010101010 **2 mark**

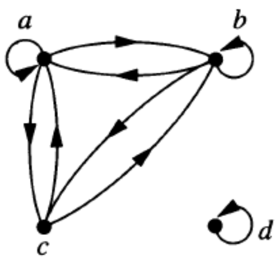
c. How may bit strings of length eight contain either three consecutive 0s or four consecutive 1s? **4 marks**

d. What is the minimum number of people required to guarantee that at least 3 share the same birthday? **2 marks**

e. List the ordered pairs represented by the following digraphs.

i. 

3 marks

ii. 

3 marks